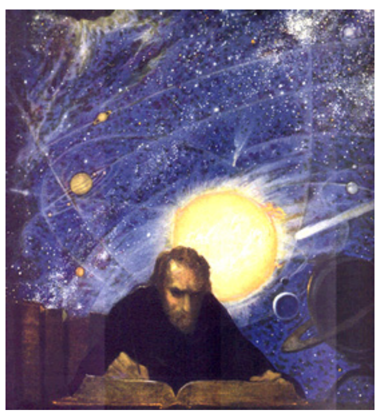
# FRESNEL-C

## Reflectance and Transmittance at the Interface of Two Media with Complex Refractive Indices

# Manual





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# CONTENT

# 1. INTRODUCTION

FRESNEL-C is the program for calculating and plotting against incidence angle Fresnelian reflectance and transmittance at the interface of two media with complex refractive indices. FRESNEL-C also calculates polarization degree of reflected and transmitted optical radiation as well as critical, Brewster's and pseudo-Brewster's angles.

Usually, Fresnel's law is written for the simplest case when the medium from which a ray falls onto two media interface, is transparent (i.e., its extinction coefficient is equal to zero or very small). However, in some instances (e.g., in photolithography, or at the studying of reflectance for paints and various coatings on the metallic substrates), it is impossible to neglect absorptance of the first medium because this might lead to incorrect physical results.

FRESNEL-C is intended for researcher working in area of optical radiometry, photometry, optical technology, and also for in-depth studying of Fresnelian reflection.

FRESNEL-C works on PC with operating system MS Windows from 2000 to 7. Screen resolution must be 1024×768 pixels or greater.

# 2. THEORETICAL BASIS

## *2.1. Electromagnetic Radiation*

Physical optics considers optical radiation as electromagnetic radiation of optical frequency range – from UV to far IR. Electromagnetic radiation is a form of energy exhibiting wave-like behavior as it propagates. Electromagnetic theory is based on Maxwell's equations according to which a time-varying electric field generates a time-varying magnetic field and so on. Such oscillating fields together form a propagating transverse electromagnetic wave which has both electric and magnetic field components, which oscillate in phase perpendicular to each other and perpendicular to the direction of energy propagation. In vacuum, electromagnetic radiation propagates at the speed of light, $c$ = 299,792,458 $m \cdot s^{-1}$; in a material medium electromagnetic radiation propagates at the speed less than *c.*

Maxwell's equations lead directly to electric field intensity *E* and magnetic field density *B* satisfying the wave equation for which the solutions are linear combinations of plane waves. We omit all mathematical and physical issues concerning derivation and solution of appropriate equations and refer interested reader to numerous books on electrodynamics, e.g. [1 – 4].

## *2.2. Polarization of Electromagnetic Waves*

Electromagnetic waves as transverse waves can be characterized by their polarization state that describes the orientation of wave oscillations. The polarization of electromagnetic waves is described by specifying the orientation of the wave's electric field intensity, *E*, at a point in space during one period of the oscillation. Oscillations in electromagnetic wave propagated through vacuum are perpendicular to the wave's propagation direction. The wave is linearly polarized if the vector **E** of electric field is oriented in a single direction. In the cases of circular and elliptical polarization **E** rotates at the time of wave propagation and can be decomposed into two mutually perpendicular components at any time of oscillation period.

## *2.3. Optical Constants of Media*

The characteristics of medium that in general use in optical region of electromagnetic spectrum are the refractive index, *n*, and the extinction coefficient, *k*. They are often called *optical constants* of material under consideration. Optical constants of media are not true constants; they vary with the frequency of optical radiation.

Refractive index of a medium is a ratio of the speed of light in vacuum to that in the medium under consideration. For dielectrics, $n > 1$; metals may have $n < 1$ for some wavelengths (see for details Ref. [5], Chapter XII). Here, we'll not consider such artificial structures as metamaterials [6] with negative refractive indices at some wavelengths.

The extinction coefficient indicates the amount of absorption loss when the electromagnetic wave propagates through the medium. For lossless medium (vacuum), $k = 0$; for other media (except special situation as leasers), $k > 0$. Positive value of *k* determines material's transparency.

A complex refractive index can be defined as $m = n + i \cdot k$, where $i^2 = -1$. There is alternative definition of a complex refractive index as $m = n - i \cdot k$, but we'll use the first definition since the convention about the sign of imaginary part of $m$ does not affect the results we'll use.

Reference data for *n* and *k* of solid materials can be found in [7]. There is also voluminous literature containing optical constants data for various specific materials.

## *2.4. Reflection and Refraction Laws*

Detailed description of physical phenomena occurring when the optical radiation intersects the interface of two different media can be found in many fundamental books on optics (see, for instance, Refs. [5, 8 – 11) In brief, plane electromagnetic wave incident onto a boundary of two homogeneous media having different optical properties is split into a transmitted wave penetrated into the second medium and reflected wave propagated back into the first medium. Generally, both components are polarized even if incident radiation is unpolarized, and their polarization degree depends on optical constants of two media as well as the incidence angle.

If the interface between two media is plane and smooth enough (this means that the wavelength is less significantly than the average roughness size), the reflection is considered as specular (or regular). Specular reflection obeys the following laws: (i) the incident ray, the reflected ray and the normal to the surface of reflection at the point of the incidence belong to the same plane; (ii) the angle between the incident ray and the normal is equal to the angle between the reflected ray and the same normal:

$$\theta_r = \theta_i \,, \tag{1}$$

where $\theta_i$ and $\theta_r$ are above-mentioned incidence and reflection angles, respectively.

The plane in which the incident ray, the normal to the surface, and the reflected ray lie is called *plane of incidence*.

When optical radiation crosses over the boundary of two media with refractive indices $n_1$ and $n_2$, the following relationship (Snell's law) takes place for incidence angle $\theta_i$ and refraction angle $\theta_t$:

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{n_2}{n_1}\,. \tag{2}$$

If to introduce relative refractive index $n = n_2/n_1$, Snell's law can be re-written in form

$$\frac{\sin\theta_i}{\sin\theta_t} = n\,. \tag{3}$$

The refraction angle can be expressed as

$$\theta_t = \arcsin\left(\frac{\sin\theta_i}{n}\right). \tag{4}$$

It is easy to see that Eq. () is valid only if $\frac{\sin\theta_i}{n} \le 1$. The critical angle $\theta_{cr}$ is the value of $\theta_i$ for which $\theta_t$ equals 90°:

$$\theta_{cr} = \arcsin n = \arcsin\left(\frac{n_2}{n_1}\right). \tag{5}$$

This means that if optical radiation propagates from the medium with the greater refractive index to the medium with the lesser refractive angle, there is the critical incidence angle $\theta_{cr}$ defining by Eq. (5) above which no radiation passes trough the boundary of media; all incident radiation will be reflected. This phenomenon is known as *total internal reflection*.

## *2.5. Fresnel Equations*

The fraction of the incident power that is reflected from the interface is given by the reflectance *R* and the fraction that is refracted is given by the transmittance *T*. The values of *R* and *T* depend on polarization of the incident radiation. The Fresnel equations allow calculating the reflectance and transmittance at a plane boundary between two different media. Here and hereinafter, we shall assume that (i) the incident wavefront is plane and (ii) both media are homogeneous and isotropic.

In many monographs, handbooks, and textbooks on optics [5, 8 - 12] it is also assumed that both media are perfectly transparent, i. e., $k_1 = k_2 = 0$. In this case, if the radiation is polarized with the vector of electric field intensity perpendicular to the plane of incidence (*s*-polarized), the reflectance $R_s$ can be calculated as:

$$R_s = \left( \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right)^2 , \tag{6}$$

or, after application of Snell's law:

$$R_s = \left[ \frac{n_1 \cos\theta_i - n_2 \sqrt{1 - \left( \frac{n_1}{n_2} \sin\theta_i \right)^2}}{n_1 \cos\theta_i + n_2 \sqrt{1 - \left( \frac{n_1}{n_2} \sin\theta_i \right)^2}} \right]^2 . \tag{7}$$

If the incident radiation is polarized in the plane of incidence (*p*-polarization) then

$$R_p = \left( \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right)^2 = \left[ \frac{n_1 \sqrt{1 - \left( \frac{n_1}{n_2} \sin\theta_i \right)^2} - n_2 \cos\theta_i}{n_1 \sqrt{1 - \left( \frac{n_1}{n_2} \sin\theta_i \right)^2} + n_2 \cos\theta_i} \right]^2 . \qquad (8)$$

If the incident radiation is unpolarized, i. e., consists of equal amount of random polarization states, the reflectance is $R = \frac{1}{2}\left(R_p + R_s\right)$. Energy conservation law leads to the following equations for transmittances:

$$T_s = 1 - R_s; \quad T_p = 1 - R_p; \quad T = 1 - R . \qquad (9)$$

For $\theta_i = 0^\circ$ we obtain well-known formula for normal reflectance:

$$R = R_p = R_s = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2 . \qquad (10)$$

As it was mentioned above, Eqs. (6) – (10) are written for the case $k_1 = k_2 = 0$ (case of lossless media). When $k_1 \neq 0$ and $k_2 \neq 0$, Eqs. (6) – (10) become invalid. Hereinafter, we'll follow mainly Ref. [13].

The geometry and notation for this general case are shown in Fig. 1. The boundary separates the first medium with the complex refractive index $m_1 = n_1 + ik_1$, from the second with $m_2 = n_2 + ik_2$. A plane wave is incident on the surface from medium 1 into medium 2.

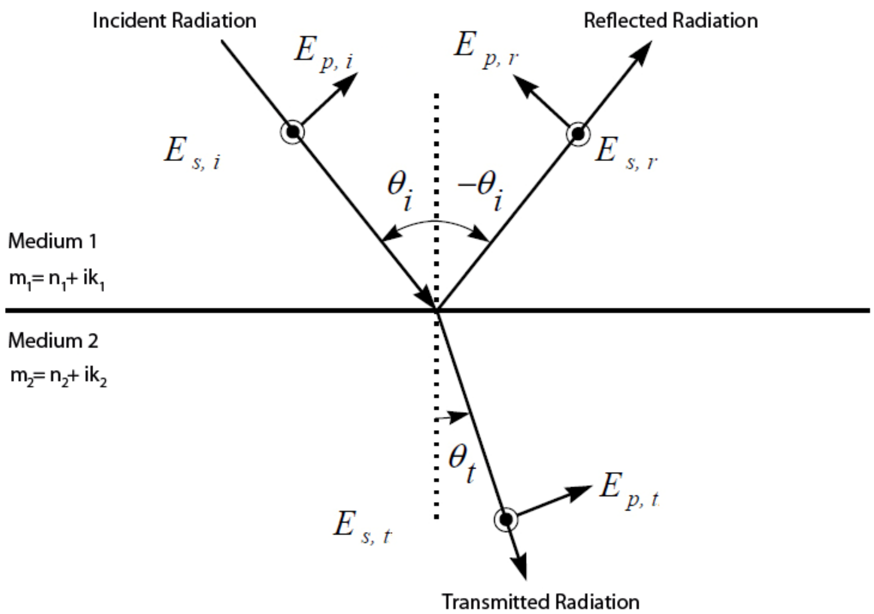



Fig. 1. Geometry and notation for reflection and transmission at the interface of two media.

By introducing $m$ , the refractive index of the first medium relative to the second,

$$m = \frac{m_2}{m_1} = n + ik \,, \tag{11}$$

one can find

$$n = \frac{n_1 n_2 + k_1 k_2}{n_1^2 + n_2^2} \tag{12}$$

and

$$k = \frac{n_1 k_2 - k_1 n_2}{n_1^2 + n_2^2} \,. \tag{12}$$

$$R_p = \frac{\left[\left(n^2 - k^2\right)\cos\theta_i - X\right]^2 + \left[2nk\cos\theta_i - Y\right]^2}{\left[\left(n^2 - k^2\right)\cos\theta_i + X\right]^2 + \left[2nk\cos\theta_i + Y\right]^2} \,, \tag{13}$$

$$R_s = \frac{(\cos\theta_i - X)^2 + Y^2}{(\cos\theta_i + X)^2 + Y^2}, \qquad (14)$$

where

$$X^2 = n^2 - k^2 - \sin^2\theta_i + \sqrt{(n^2 - k^2 - \sin^2\theta_i)^2 + 4n^2k^2}, \qquad (15)$$

$$Y^2 = k^2 + \sin^2\theta_i - n^2 + \sqrt{(n^2 - k^2 - \sin^2\theta_i)^2 + 4n^2k^2}, \qquad (16)$$

The sign of $\sqrt{Y^2}$ in Eq. (13) should be chosen to provide correct transition from Eq. (13) to Eq. (8) for $k \to 0$.

If $R_p = 0$ (this is possible if and only if $k = 0$, i. e., when both media are lossless ), reflected radiation is linearly polarized in the plane perpendicular to the plane of incidence. Angle for which this occurs is called *Brewster's angle*. It has a simple expression:

$$\theta_B = \arctan\frac{n_2}{n_1}. \qquad (17)$$

If $k_1 > 0$ or $k_2 > 0$, $R_p$ also reaches minimum, but its value remains positive. In such a case, we deal with the *pseudo-Brewster angle*. It can be found from the solution of a cubic equation in $\sin^2\theta_i$ [14].

The *polarization degree* of reflected and transmitted radiation are defined as

$$P_r = 100\% \cdot \frac{R_s - R_p}{R_s + R_p} \qquad (18)$$

and

$$P_t = 100\% \cdot \frac{T_s - T_p}{T_s + T_p}, \qquad (19)$$

respectively.

$P_r$ is always positive and becomes +100% at the Brewster angle. $P_t$ can be negative but never reaches -100%. The sign of polarization degree define the direction of the wave phase shift that occurs after reflection and transmission.

# 3. WORKING WITH FRESNEL-C

## *3.1. Installation of FRESNEL-C*

FRESNEL-C does not require special efforts for installation. Simply download the Evaluation version of the program from www.virial.com and unzip fresnel_c.zip to any place of the hard drive and run fresnel_c.exe. The following window will appear:

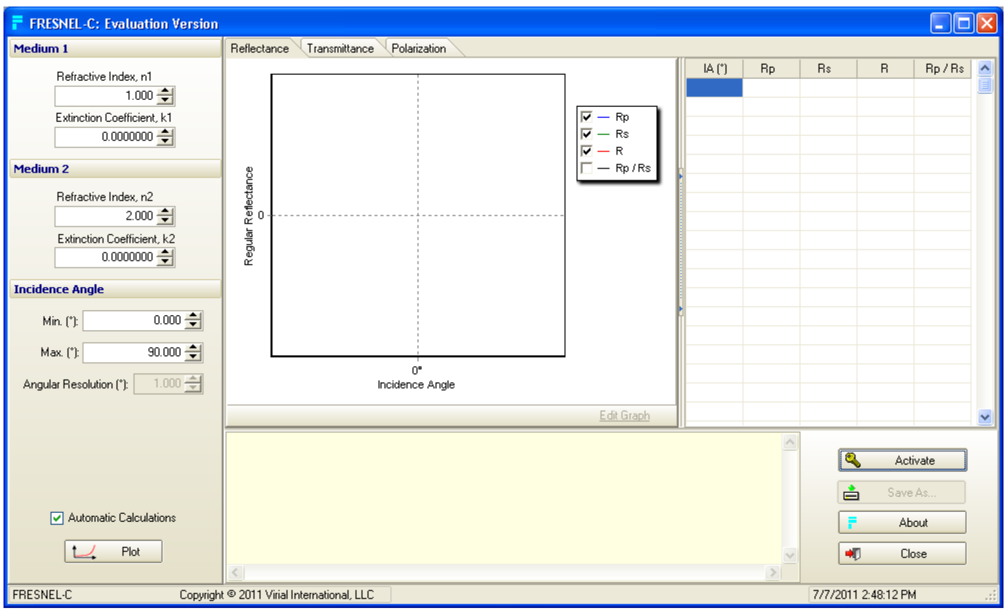



Fig. 2. FRESNEL-C before activation (Evaluation Version)

Press  ; the Activation window will appear:


Fig. 3. Activation window

Enter the activation key then press OK. This turns Evaluation version into full-functioned program:

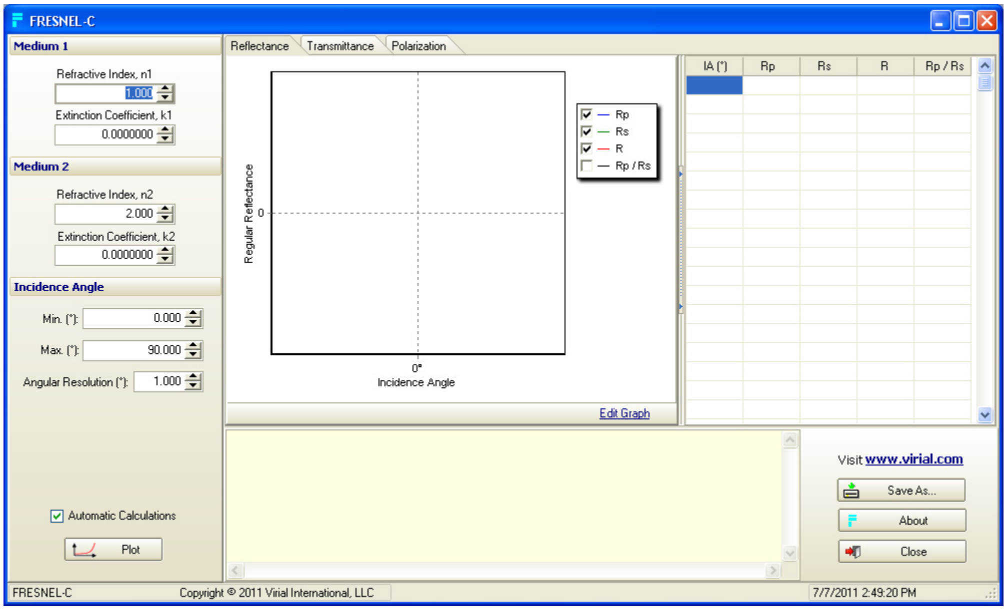



Fig. 4. FRESNEL-C after activation (full-functioned program)

## *3.2. Calculations with FRESNEL-C*

There are two modes of calculation and graph plotting using FRESNEL-C:

- Non-automatic calculations that available always by pressing Plot and allow to fill the tables and plots dependences on incidence angles and
- Automatic calculations that are performed in real time as soon as initial values are changed. This mode is available only if checkbox Automatic Calculations is checked and angular resolution is not less than 0.1º; if angular resolution is less than 0.1º, the appropriate checkbox is unchecked and disabled: Automatic Calculations.

Input of initial values can be done using keyboard or spinners attached to editable fields and increasing/decreasing numerical values incrementally. Initial values are:

- Refractive Index, *n1*, of the first medium from which radiation falls onto boundary,
- Extinction Coefficient, *k1*, of the first medium,
- Refractive Index, *n2*, of the second medium,

- Extinction Coefficient, *k2*, of the second medium,
- Min. and Max. Incidence Angles (in º), and
- Angular Resolution (in º).

Calculations are performed for each incidence angle from Min. to Max. with an increment equals angular resolution for the following functions:

1. Reflectance $R_p$ for *p* polarization state,
2. Reflectance $R_s$ for *s* polarization state,
3. Reflectance *R* for unpolarized radiation,
4. Ratio $R_p/R_s$ ,
5. Transmittance $T_p$ for *p* polarization state,
6. Transmittance $T_s$ for *s* polarization state,
7. Transmittance *T* for unpolarized radiation,
8. Polarization Degree PR for reflected radiation, and
9. Polarization Degree PT for transmitted radiation.

All these functions are represented in graphs and in tabular form (see Figs. 5 – 7), each on the separate tabbed page. The content of each table together with the initial data and such an additional information as critical, Brewster and pseudo-Brewster angles, etc. displayed under the graph can be saved as a text file after pressing Save As... . A sample of such a text file is presented below.

```
n1 = 1.900000, k1 = 0;   n2 = 3.40000, k2 = 1.4;   91 points
Relative Complex Refrative Index = 1.78947 + i * 0.73684211
Pseudo-Brewster Angle (PBA) = 61.846°
Rp @ PBA = 0.02382
Rs @ PBA = 0.392605
(Rp / Rs) @ PBA = 0.06067
Angle (°)  Rs       Rp       R
 0.000000 0.140100 0.140100 0.140100
 1.000000 0.140059 0.140141 0.140100
 2.000000 0.139937 0.140263 0.140100
 3.000000 0.139733 0.140467 0.140100
 4.000000 0.139447 0.140754 0.140100
 5.000000 0.139080 0.141123 0.140101
 6.000000 0.138631 0.141575 0.140103
 7.000000 0.138101 0.142111 0.140106
 8.000000 0.137489 0.142732 0.140110
 9.000000 0.136795 0.143438 0.140116
10.000000 0.136019 0.144231 0.140125
……………………………………………………………………………………………………

80.000000 0.232908 0.708627 0.470767
81.000000 0.270883 0.733234 0.502058
82.000000 0.314542 0.758769 0.536656
83.000000 0.364698 0.785261 0.574980
84.000000 0.422292 0.812740 0.617516
85.000000 0.488417 0.841234 0.664825
86.000000 0.564347 0.870774 0.717560
```

87.000000 0.651568 0.901390 0.776479
88.000000 0.751825 0.933112 0.842468
89.000000 0.867164 0.965972 0.916568
90.000000 1.000000 1.000000 1.000000

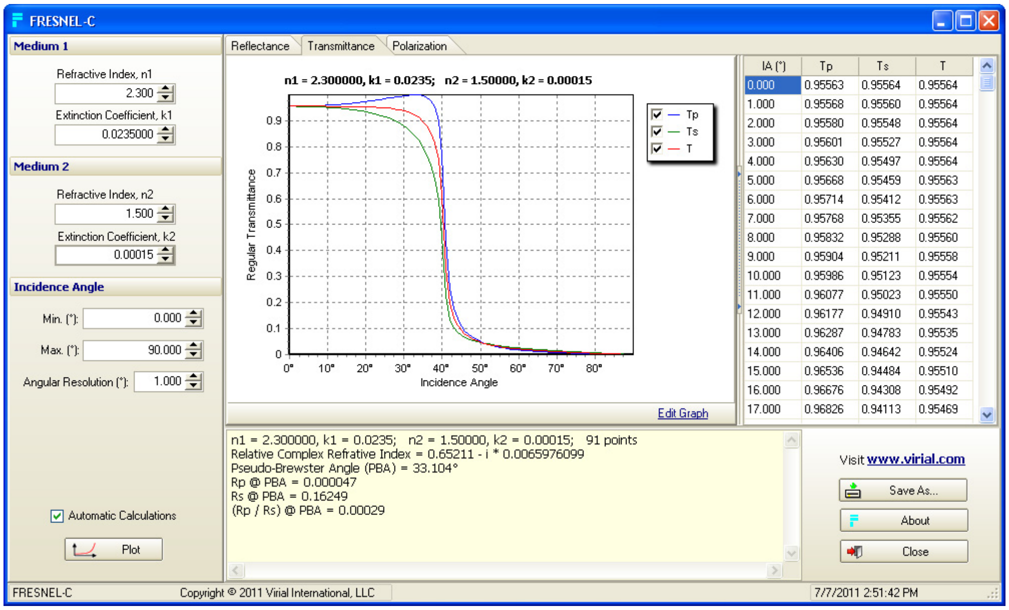



Fig. 5. Tabbed page “Reflectance”

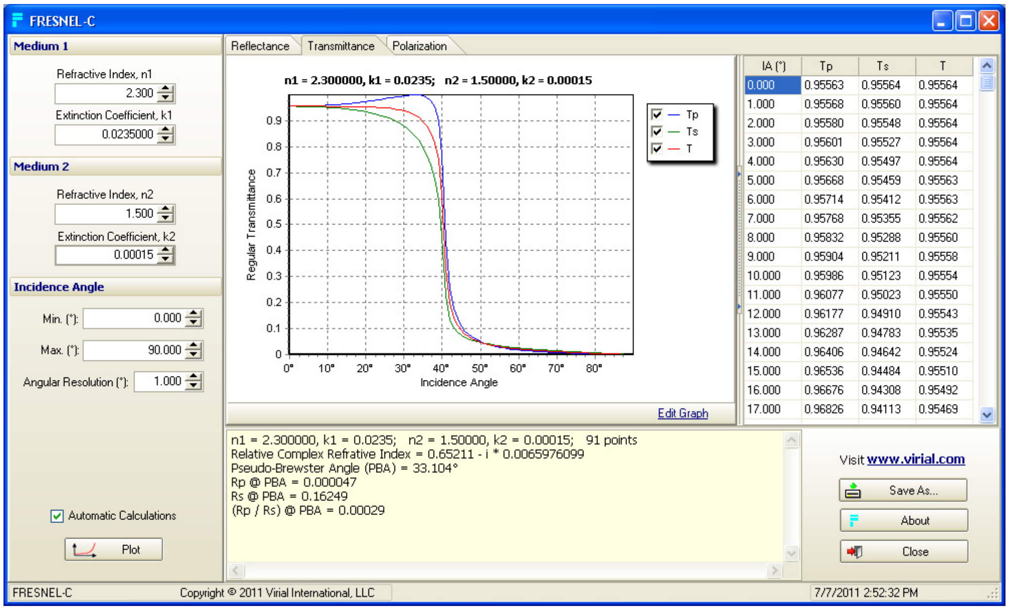


Fig. 6. Tabbed page “Transmittance”

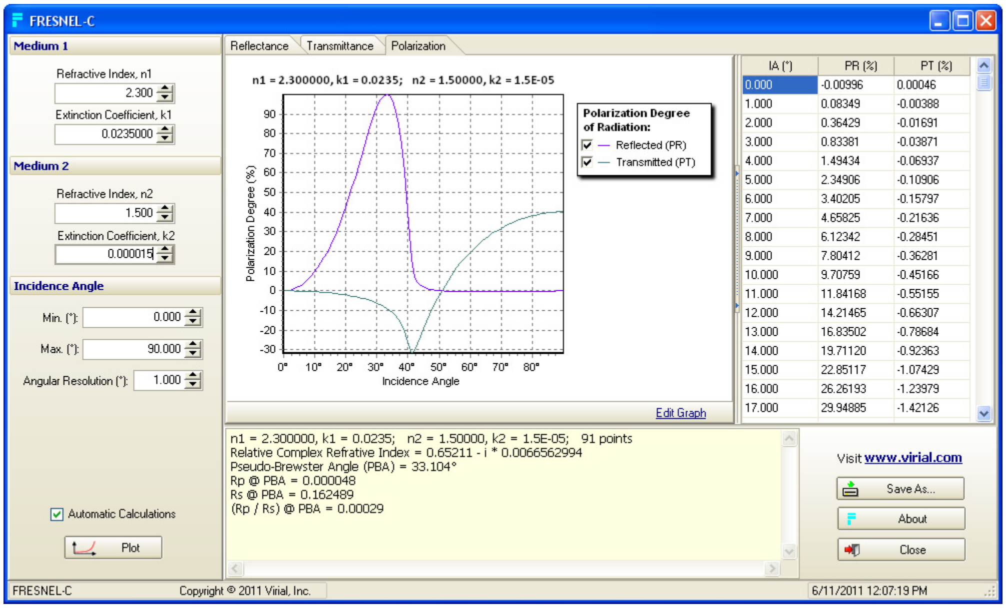



Fig. 7. Tabbed page “Polarization”

## *3.3. Working with Graphs*

FRESNEL-C allows plotting a magnified fragment of the graph: holding left mouse button depressed, drag the cursor right and downwards to zoom (see Fig. 8) and left and upwards to unzoom.

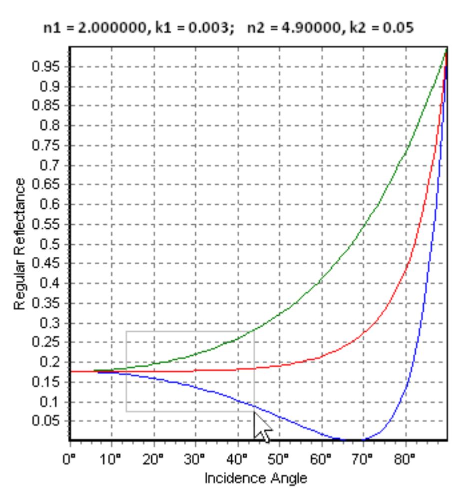
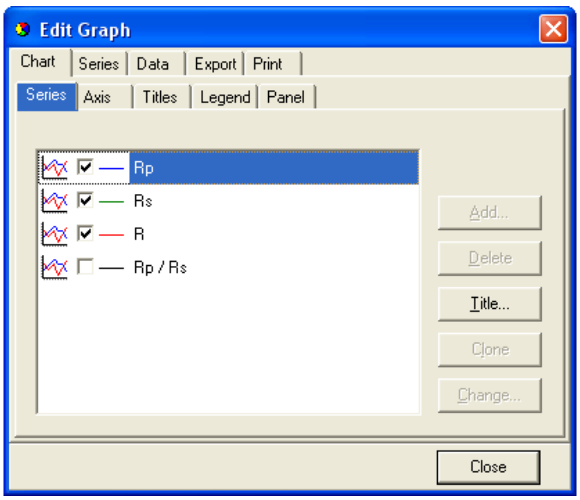


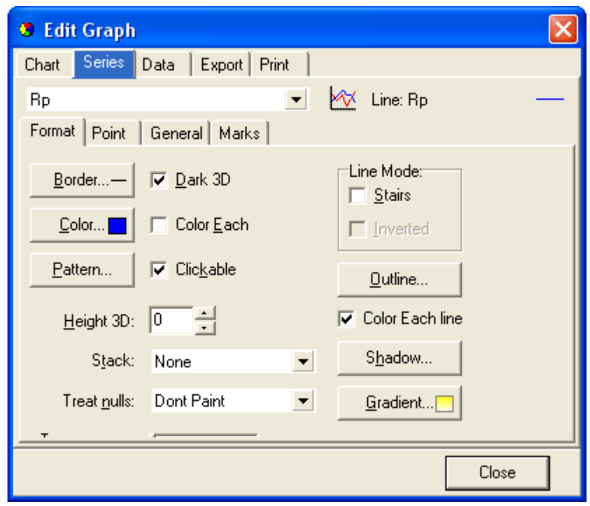
Fig. 8. Use of zoom

To displace curves relative to graph axes, hold the left mouse button depressed and move cursor. To restore graph original position, draw a rectangle of arbitrary size by moving from the bottom right corner to the top left one while left mouse button remains pressed.

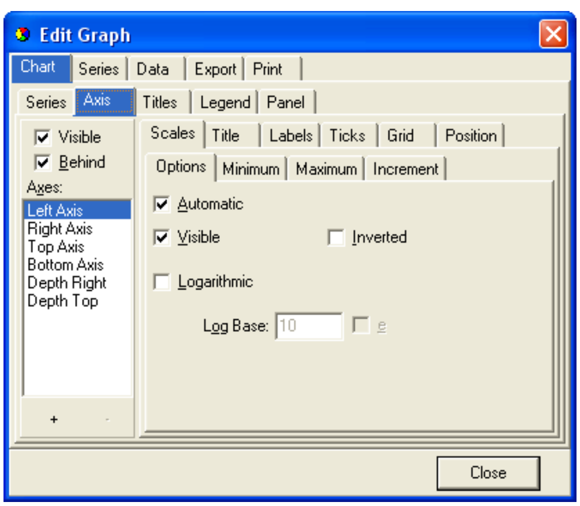
Click the link Edit Graph below the graph which should be edited to call the Graph Editor that has intuitive interface and provides comprehensive access to the most of the editable properties of each graph (see Figs. 9A and 9B).



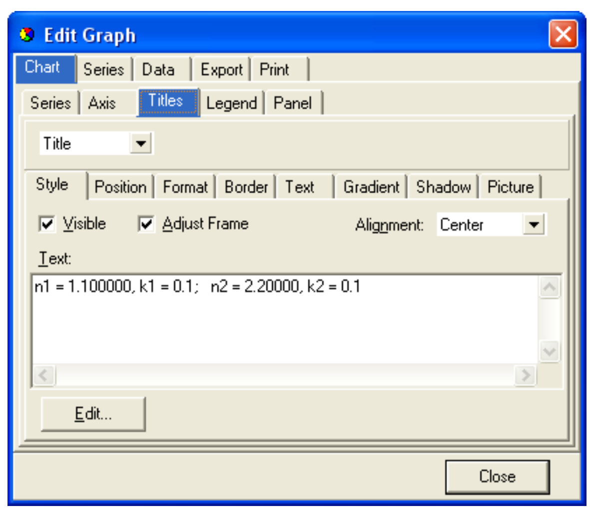

Access to individual series (curves)




Formatting the series



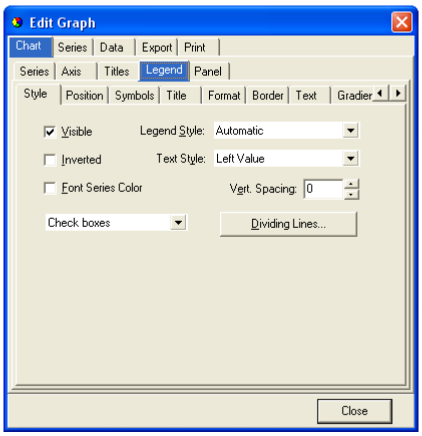

Editing the graph axes



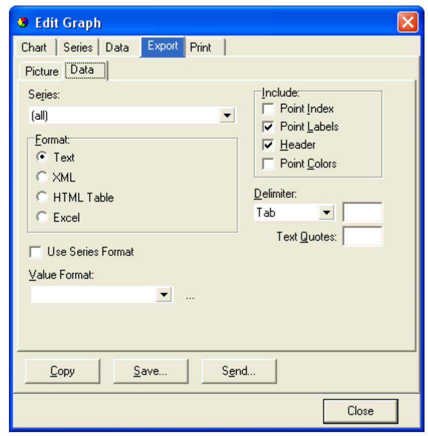

Editing the title

Fig. 9A. Screenshots that demonstrate general possibilities of the Graph Editor

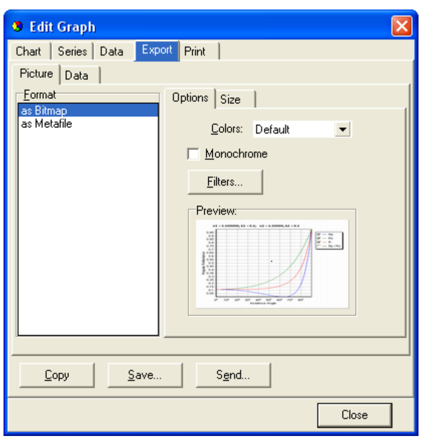
Graph Editor provides access to individual curves (series) allowing to edit all elements of the graph (points, axes, legend, title, etc.) and adjust their properties. The Graph Editor gives the possibility of copying to clipboard, saving in the file, and printing graphs, as well as exporting series values in formats of text (ASCII) file, MS Excel spreadsheet, HTML and XML tables.



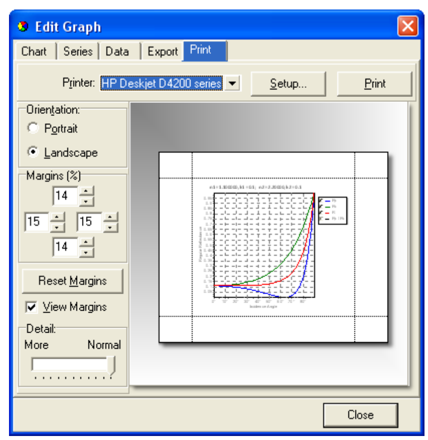

Formatting the graph legend




Data Export




Graph copying and saving




Graph printing

Fig. 9B. Screenshots that demonstrate general possibilities of the Graph Editor

# 4. EVALUATION VERSION VS. FULL-FUNCTIONED PROGRAM

FRESNEL-C will work in Evaluation mode until you activated it by entering activation key you'll obtain as soon as the license will be purchased. Evaluation Version of FRESNEL-C uses the same computational procedures as the full-functioned program but has several restrictions:

1. Evaluation version doesn't allow to enter initial values using keyboard; they can be changed only using spinners; all initial values are integer; increment is equal to 1.
2. Angular resolution is equal to 1º and cannot be changed.
3. Content of tables and auxiliary information cannot be saved.
4. Graphs cannot be edited.

Procedure of activation FRESNEL-C is described in Section 3.1.

## 5. REFERENCES

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16. The Vendor will be free of liability to the Licensee where the Vendor is prevented from executing its obligations under this Agreement in whole or in part due to Force Majeure, such as earthquake, typhoon, flood, fire, and war or any other unforeseen and uncontrollable event where the Vendor has taken any and all appropriate action to mitigate such an event.

**Governing Law**

17. The Parties to this Agreement submit to the jurisdiction of the courts of the State of Maryland for the enforcement of this Agreement or any arbitration award or decision

arising from this Agreement. This Agreement will be enforced or construed according to the laws of the State of Maryland.

**Miscellaneous**

18. This Agreement can only be modified in writing signed by both the Vendor and the Licensee.

19. This Agreement does not create or imply any relationship in agency or partnership between the Vendor and the Licensee.

20. Headings are inserted for the convenience of the parties only and are not to be considered when interpreting this Agreement. Words in the singular mean and include the plural and vice versa. Words in the masculine gender include the feminine gender and vice versa. Words in the neuter gender include the masculine gender and the feminine gender and vice versa.

21. If any term, covenant, condition or provision of this Agreement is held by a court of competent jurisdiction to be invalid, void or unenforceable, it is the parties' intent that such provision be reduced in scope by the court only to the extent deemed necessary by that court to render the provision reasonable and enforceable and the remainder of the provisions of this Agreement will in no way be affected, impaired or invalidated as a result.

22. This Agreement contains the entire agreement between the parties. All understandings have been included in this Agreement. Representations which may have been made by any party to this Agreement may in some way be inconsistent with this final written Agreement. All such statements are declared to be of no value in this Agreement. Only the written terms of this Agreement will bind the parties.

23. This Agreement and the terms and conditions contained in this Agreement apply to and are binding upon the Vendor's successors and assigns.

**Notices**

24. All notices to the Vendor under this Agreement are to be provided at the following address:

Virial International, LLC
538 Palmspring Dr.,
Gaithersburg, MD 20878-2972
USA