A source of optical radiation with characteristics calculable on the basis of fundamental physical laws makes possible calibration of radiometers, spectroradiometers, radiation thermometers, and other radiometric equipment. From theoretical point of view, a perfect blackbody is the most suitable object for this purpose. If its thermodynamic temperature is known, spectral characteristics of blackbody radiation can be computed using Planck's law; the Stefan-Boltzmann law determines total radiation characteristics. However, a perfect blackbody is a physical abstraction that does not exist in real world. The perfect blackbody conditions are approximately realized inside an isothermal cavity with opaque walls. Radiation escaping a cavity through a tiny opening very closely imitates radiation of a perfect blackbody. In order to employ an artificial blackbody as a standard reference source, it is necessary to know how large are differences in radiation characteristics of a cavity and those of a perfect blackbody due to geometry of a cavity, actual temperatures of its walls and optical properties of their material.

There are two different objects referred in literature as “blackbody”:

1. A theoretical object that completely absorbs all radiation incident upon it. Blackbody emits maximal amount of radiant energy at given wavelength and given temperature in comparison with all other radiating bodies.
2. An artificial source of optical radiation designed to simulate characteristics of a perfect blackbody and used as a reference source with calculable radiation characteristics.

In order to differentiate them, we shall use the term “perfect blackbody” for a theoretical object, keeping the term “blackbody” for an artificial source.

Quantitative measure of the difference in radiation characteristics between an artificial blackbody and a perfect blackbody is the effective emissivity. The qualifier “effective” is used due to the effect produced by multiple reflections. Unlike a flat sample, outgoing radiation of an element of a cavity wall consists not only of its own thermal radiation, but also of radiation falling from other surface elements and reflected by the element under consideration. The effective emissivity is determined by the cavity geometry, optical properties of the cavity walls, viewing conditions, i.e., geometry of collecting the cavity radiation, and the temperature distribution over the radiating surface.

Generally speaking, effective emissivity is the ratio of a radiometric quantity (usually, radiance or spectral radiance) that characterizes a blackbody radiator at a certain temperature to the same quantity of a perfect blackbody with the same temperature. However, real-world cavities are always nonisothermal. To avoid ambiguity in temperature assigned to the perfect blackbody in the effective emissivity definition, the reference temperature $T_{\text{ref}}$ has to be introduced and assigned to the perfect blackbody. $T_{\text{ref}}$ itself has no specific physical meaning; its choice, in general, is arbitrary. Depending on the $T_{\text{ref}}$ choice, the spectral effective emissivity of a nonisothermal blackbody can take any positive value, even be greater than unity. This means that the spectral radiance of a nonisothermal source is greater than that of a perfect blackbody at the reference temperature, the same wavelength, and under other equal conditions. If some areas of radiating surface has temperatures greater than $T_{\text{ref}}$, the
effective emissivity can exceed unity. Practically, in order to keep effective emissivity of the nonisothermal blackbody comparable with those for the isothermal one, the temperature measured by a contact temperature sensor is commonly used for $T_{\text{ref}}$. Effective emissivity of a nonisothermal cavity depends on the wavelength even if the cavity internal surface has wavelength-independent optical characteristics.

The quantities characterizing blackbody radiation sources are usually defined for the non-refracting, non-absorbing, non-scattering, and non-emitting environment (i.e., vacuum at 0 K). It is also supposed that optical properties of cavity walls do not depend on temperature. The effect of background radiation will be considered later.

**Spectral Local Directional Effective Emissivity**

The primary characteristic of an artificial blackbody is the spectral local directional effective emissivity $\varepsilon_e$. It is defined by the following equation:

$$
\varepsilon_e(\lambda, \xi, \omega, T_{\text{ref}}) = \frac{L_\lambda(\lambda, \xi, \omega)}{L_{\lambda,\text{bb}}(\lambda, T_{\text{ref}})},
$$

where $L_\lambda$ is spectral radiance (in W·m⁻³·sr⁻¹) of the radiation coming from a point on blackbody wall at a particular wavelength $\lambda$, with coordinates specified by the vector $\xi$, and the direction in which the radiation is emitted specified by the vector $\omega$; $L_{\lambda,\text{bb}}$ is spectral radiance of a perfect blackbody at a reference temperature $T_{\text{ref}}$ and the same wavelength $\lambda$.

The numerator in Eq. (1) refers to the sum of own thermal radiation of the surface element and radiation that is falling from all possible directions and is reflected by this element in the direction $\omega$. Denominator in Eq. (1) is expressed by the Planck law:

$$
L_{\lambda,\text{bb}}(\lambda, T_{\text{ref}}) = \frac{c_1}{\pi \cdot \lambda^5} \exp\left(\frac{c_2}{\lambda \cdot T_{\text{ref}}} - 1\right),
$$

where $c_1 = 3.74177153 \times 10^{-16}$ W·m² and $c_2 = 1.4387770 \times 10^{-2}$ m·K are the 1st and 2nd radiation constants, respectively [1].
Radiation characteristics of a blackbody with inhomogeneous temperature can differ significantly from those of the isothermal blackbody. As it was already mentioned, the effective emissivity of a nonisothermal radiator can take any positive value, depending on the reference temperature choice. However, the spectral radiances of a blackbody do not change their values at any choice of $T_{\text{ref}}$. If $T_{\text{ref}}$ and $T'_{\text{ref}}$ are two reference temperatures, then

$$
\varepsilon_e (\lambda, \xi, \omega, T_{\text{ref}}) = \frac{\exp \left( \frac{c_2}{\lambda T_{\text{ref}}} \right) - 1}{\exp \left( \frac{c_2}{\lambda T'_{\text{ref}}} \right) - 1}.
$$

(3)

**Bandlimited and Total Local Directional Effective Emissivities**

Integration of Eq. (1) over the entire spectrum together with the relative spectral responsivity $r(\lambda)$ of a detector, gives the bandlimited local directional effective emissivity:

$$
\bar{\varepsilon}_e (\lambda, \xi, \omega, T_{\text{ref}}) = \frac{\int_0^\infty r(\lambda)L_\lambda (\lambda, \xi, \omega) d\lambda}{\int_0^\infty r(\lambda)L_{\lambda, bb} (\lambda, \xi, \omega, T_{\text{ref}}) d\lambda}.
$$

(4)

Integration over the entire spectrum for $r(\lambda) \equiv 1$ reduces Eq. (4) to a ratio of radiances. The Stefan-Boltzmann law allows defining total local directional effective emissivity:

$$
\varepsilon_e (\xi, \omega, T_{\text{ref}}) = \frac{\pi L(\xi, \omega)}{\sigma T_{\text{ref}}^4},
$$

(5)

where $L$ is radiance (in W·m$^{-2}$·sr$^{-1}$) of the cavity wall and $\sigma = 5.670400 \times 10^{-8}$ W·m$^{-2}$·K$^{-4}$ is the Stefan-Boltzmann constant [1].

**Hemispherical and Integrated Effective Emissivities**

Integration of Eq. (1) over a hemispherical solid angle transforms the spectral local directional effective emissivity $\varepsilon_e (\lambda, \xi, \omega, T_{\text{ref}})$ to the spectral hemispherical effective emissivity $\varepsilon_{e,h} (\lambda, \xi, T_{\text{ref}})$ and the term for spectral radiance $L_\lambda$ to that for spectral radiant exitance $M_\lambda$. According to Lambert’s law, spectral radiance of the perfect blackbody does not depend on the angle of observation and can be expressed through the spectral exitance $M_\lambda$ (surface density of the emitted radiant power):

$$
L_{\lambda, bb} (\lambda, T) = \frac{1}{\pi} M_{\lambda, bb} (\lambda, T).
$$

(6)
The spectral, bandlimited, and total hemispherical effective emissivities are defined by the following three equations:

\[
\varepsilon_{e,h}(\lambda, \Xi, T_{\text{ref}}) = \frac{M_{\lambda}(\lambda, \Xi)}{M_{\lambda,bb}(\lambda, T_{\text{ref}})} = \frac{M_{\lambda}(\lambda, \Xi)}{\pi L_{\lambda,bb}(\lambda, T_{\text{ref}})},
\]

(7)

\[
\bar{\varepsilon}_{e,h}(\lambda, \Xi, T_{\text{ref}}) = \frac{1}{\int_{0}^{\infty} r(\lambda)M_{\lambda,bb}(\lambda, T_{\text{ref}}) d\lambda} \int_{0}^{\infty} r(\lambda)M_{\lambda}(\lambda, \Xi) d\lambda = \frac{1}{\pi \int_{0}^{\infty} r(\lambda)L_{\lambda,bb}(\lambda, T_{\text{ref}}) d\lambda}.
\]

(8)

\[
\varepsilon_{e,c}(\lambda, T_{\text{ref}}) = \frac{M(\Xi)}{M_{bb}(T_{\text{ref}})} = \frac{M(\Xi)}{\sigma T_{\text{ref}}^4}.
\]

(9)

Often, it is needed to know the spectral integrated effective emissivity. This quantity, \(\varepsilon_{c,e}\), is the ratio of the spectral radiant flux \(\Phi_{\lambda}\) falling onto the detector from a blackbody to the spectral radiant flux \(\Phi_{\lambda,bb}\) falling from a perfectly black surface, that replaces a blackbody aperture and has the temperature \(T_{\text{ref}}\):

\[
\varepsilon_{c,e}(\lambda, T_{\text{ref}}) = \frac{\Phi_{\lambda}(\lambda)}{\Phi_{\lambda,bb}(\lambda, T_{\text{ref}})}.
\]

(10)

The bandlimited and total integrated effective emissivities can be expressed as:

\[
\bar{\varepsilon}_{e,c}(\lambda, T_{\text{ref}}) = \frac{\int_{0}^{\infty} r(\lambda)\Phi_{\lambda}(\lambda) d\lambda}{\int_{0}^{\infty} r(\lambda)\Phi_{\lambda,bb}(\lambda, T_{\text{ref}}) d\lambda}.
\]

(11)

\[
\varepsilon_{e,c}(T_{\text{ref}}) = \frac{\Phi}{\Phi_{bb}(T_{\text{ref}})}.
\]

(12)

Depending on particular viewing conditions used for various types of radiometers, radiation thermometers, etc., one can define appropriate types of effective emissivities by averaging local directional effective emissivity over a visible part of cavity’s internal surface and an appropriate solid angle.
Effect of Background Radiation

All previous definitions have been done for a non-radiating background environment while the real environment may have temperature greater than absolute zero. Thermal radiation from the surrounding environment falls onto the aperture of a blackbody and can irradiate detector after multiple reflections inside the cavity. The simplest case of isotropic radiation of a perfect blackbody with the background temperature $T_{bg}$ is usually considered. The effect of background radiation is taken into account by the second term in the Eq. (13):

$$\varepsilon_e(\lambda, \xi, \omega, T_{ref}, T_{bg}) = \frac{\exp \left( \frac{c_2}{\lambda T_{ref}} \right) - 1}{\exp \left( \frac{c_2}{\lambda T_{bg}} \right) - 1} \varepsilon_e(\lambda, \xi, \omega, T_{ref}) + \left[ 1 - \varepsilon_e(\lambda, \xi, \omega) \right] \right] \right] \frac{\exp \left( \frac{c_2}{\lambda T_{ref}} \right) - 1}{\exp \left( \frac{c_2}{\lambda T_{bg}} \right) - 1}$$

(13)

where $\varepsilon_e(\lambda, \xi, \omega, T_{ref}, T_{bg})$ is spectral effective emissivity of a nonisothermal blackbody taking into account background radiation; $\varepsilon_e(\lambda, \xi, \omega, T_{ref})$ does not include this correction; $\varepsilon_e(\lambda, \xi, \omega)$ is spectral effective emissivity of an isothermal blackbody.

The bandlimited and total effective emissivities of a nonisothermal blackbody with the account of the background radiation can be defined by equations:

$$\bar{\varepsilon}_e(\lambda, \xi, \omega, T_{ref}, T_{bg}) = \int_0^\infty \frac{r(\lambda)\varepsilon_e(\lambda, \xi, \omega, T_{ref}, T_{bg})d\lambda}{r(\lambda)\varepsilon_e(\lambda, \xi, \omega, T_{ref}, T_{bg})L_{\lambda, bb}(\lambda, T_{ref})d\lambda}$$

$$\bar{\varepsilon}_e(\xi, \omega, T_{bg}) = \frac{\int_0^\infty \frac{r(\lambda)[1 - \varepsilon_e(\lambda, \xi, \omega)]}{\exp \left( \frac{c_2}{\lambda T_{ref}} \right) - 1} \exp \left( \frac{c_2}{\lambda T_{bg}} \right) - 1}$$

$$\varepsilon_e(\xi, \omega, T_{ref}, T_{bg}) = \varepsilon_e(\xi, \omega, T_{ref}) + \left[ 1 - \varepsilon_e(\xi, \omega) \right] \left( \frac{T_{bg}}{T_{ref}} \right)^4$$

(14)

(15)

Correction for the background radiation can be neglected if $T_{bg} \ll T_{ref}$. 
Radiometric Temperatures

Radiance temperature $T_S$ is defined [2] as temperature of a perfect blackbody, for which the spectral radiance at the given wavelength $\lambda$ has the same value as for thermal radiator under consideration. Radiance temperature is sometimes called brightness temperature in such areas as remote sensing, astrophysics, etc. The radiance temperature $T_S$ is defined by the equation:

$$L_\lambda(\lambda, \xi, \omega) = L_{\lambda,bb}(\lambda, T_S).$$  \hspace{1cm} (16)

For an artificial blackbody, the radiance temperature can be expressed through the spectral effective emissivity:

$$T_S(\lambda, \xi, \omega) = c_2 \left\{ \ln \left[ \frac{c_2}{\lambda T_{ref}} \exp \left( \frac{c_2}{\lambda T_{ref}} \right) - 1 \right] + \frac{1}{\epsilon_\omega(\lambda, \xi, \omega, T_{ref})} \right\}^{-1}.  \hspace{1cm} (17)

Eqs. (16) and (17) are also written for zero background radiation. Note that the radice temperature does not depend on $T_{ref}$ but depends on wavelength and viewing conditions.

Radiation temperature $T_R$ is defined [2] via the Stefan-Boltzmann law:

$$T_R(\xi, \omega) = T_{ref} \sqrt[4]{\epsilon_\omega(\xi, \omega, T_{ref})}.  \hspace{1cm} (11)$$

The practical radiation thermometers do not measure the radiance (monochromatic), or the radiation (total) temperature. If $r(\lambda)$ is the relative spectral responsivity of the radiation thermometer ($r(\lambda)$ is determined by the spectral transmittance of optical components, spectral sensitivity of the detector, and by other factors), certain “effective” values will be registered. The bandlimited radiance temperature $T_S$ can be found from the equation

$$\int_{0}^{\infty} r(\lambda) \epsilon_\omega(\lambda, T_{ref}) L_{\lambda,bb}(\lambda, T_{ref}) d\lambda = \int_{0}^{\infty} r(\lambda) L_{\lambda,bb}(\lambda, T_S) d\lambda,  \hspace{1cm} (12)$$

which has be solved numerically.

It should be noted that terms and definitions of bandlimited values are absent in standards and regulations [2, 3] as well as in other normative documents. This fact does not prevent researchers to use them (see, e.g., [4, 5]).
References